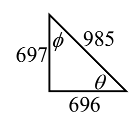
Special Pythagorean Triples

Do work and write answers on notebook paper.

Formulas for use in these exercises: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$.

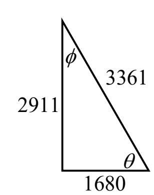
1. Suppose $m = 4$ and $n = 1$. Find the **ordered triple** (a, b, c) using the above formulas.
2. Suppose $(m, n) = (8, 3)$. Find (a, b, c). Also find $\sqrt{a^2 + b^2}$.
3. For $(m, n) = (3, 1)$, find (a, b, c).
4. The answers to the above exercises may be checked by seeing if $a^2 + b^2 = c^2$. In each exercise, (a, b, c) should be a Pythagorean triple. In the answer to the previous exercise, what is the largest factor common to a, b, & c?
5. For $(m, n) = (6, 3)$, find (a, b, c).
6. The triples found in #1 & #2 should be **primitive** Pythagorean triples. The answer to #3 is not primitive since the numbers in the triple have a common factor greater than 1. If a triple is divided by the greatest common factor of the numbers in the triple, then the resulting triple is primitive. Divide the triple from the previous exercise by the largest factor common to a, b, & c and write the resulting primitive triple.
7. Find the *four* primitive triples generated by these values of (m, n): (2, 1), (3, 2), (4, 3), & (5, 4). No work needs to be shown.
8. Consider this sequence: 1, 2, 5, 12, 29, … To find the next number in the sequence, double the last number and add the previous number. E.g., $29 = 2 \cdot 12 + 5$. Find the next *four* numbers in the sequence.
9. The sequence mentioned in the previous exercise is called the **Pell** sequence. Find the *four* primitive Pythagorean triples produced using these values of (m, n): (2, 1), (5, 2), (12, 5), & (29, 12). Work doesn't need to be shown.
10. Evaluate each expression to *four* decimal places: $985 \div 696.5$, $985 \div 408$, $\sqrt{2}$, $1+\sqrt{2}$.
11. For the triangle to the right, the value of θ (theta) is $\tan^{-1}(697 \div 696)$. Use a calculator to find θ to *two* decimal places. Use the correct label (°) when writing the degree measure of an angle.



12. The value of ϕ (phi) is $\tan^{-1}(696 \div 697)$. Find ϕ to *two* decimal places. Check work by seeing if $\theta + \phi = 90°$.
13. Call this sequence the σ (sigma) sequence: 1, 4, 15, 56, 209, … Observe that $209 = 4 \cdot 56 - 15$. To find the next number in the σ sequence, multiply the last number by 4 and subtract the previous number. The *italicized* numbers in the table below are from the σ sequence. Find the next *two* numbers after 209 in the σ sequence.
14. Call this sequence the τ (tau) sequence: 2, 7, 26, 97, … The **bold** numbers in the table are from the τ sequence. Find the next *three* numbers after 97 in the τ sequence by using the same method as in the previous exercise (i.e., quadruple the last number and subtract the previous number).

m	n
2	*1*
4	*1*
7	*4*
15	*4*
26	*15*
56	*15*

15. Each row of the 2-column table is an ordered pair of m & n. For each row, find the primitive Pythagorean triple determined by that row. Write the resulting *six* primitive Pythagorean triples as a 3-column table of numbers. No work needs to be shown.
16. Evaluate each expression to *three* decimal places: $\frac{451}{901}, \frac{1680}{3361}, \frac{780}{451}, \frac{2911}{1680}, \sqrt{3}, \frac{2911}{780}, 2+\sqrt{3}, \frac{26}{15}, \frac{56}{15}$.



17. For the triangle to the right, $\theta = \tan^{-1}(2911 \div 1680)$. Find θ to *two* decimal places. Also find ϕ to *two* decimal places (for assistance, study #12).